

**Final Report**  
**NAG5-1132**  
**Mantle Dynamics and Geodesy**  
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### **Background**

During the past years, under NASA Geodynamics sponsorship, former Caltech Professor Bradford H. Hager has made substantial progress in integrating geodesy, geodynamics, and seismology, developing the field of the interpretation of observed geoid anomalies in terms of mantle convection. Models of mantle heterogeneity inferred from seismic tomography are used as input to fluid mechanical models of mantle flow. The resulting flow leads to dynamic topography, which has a large effect on the geoid. The dynamic topography and geoid calculated for a given density model are strongly dependent on the viscosity and compositional variation with depth used in the flow model. By comparing predicted and observed geoids, we can place strong constraints on mantle structure.

In August, 1988, while he was still at the California Institute of Technology (Caltech), Hager submitted a three-year proposal to NASA Geodynamics entitled "Mantle Dynamics and Geodesy," to further develop this work. That proposal was accepted and funded (in full), with the initial year's funding for the period May 15, 1989-May 14, 1990 (NAG5-1132). Hager left Caltech to take an appointment at MIT starting July 1, 1989. He left the grant at Caltech, with Arden Albee, Dean of Graduate Students, as PI, in order to provide sufficient support for graduate student Scott King to complete his Ph. D. dissertation. (King plans to defend September 4, 1990.)

Hager is now proposing to bring this grant support to MIT. This document is a final report covering the work done under the grant to Caltech. It consists of a progress report for work completed or in progress from the initial year's funding, followed by a list of publications.

### **Progress Report**

The original proposal presented a three-year research plan. Some of the projects proposed have been completed, with publications submitted. Since efficient research does not necessarily coincide with the annual report cycle, many more projects are well underway, but not yet ready for publication. In particular, graduate student Scott King should have several additional publications originating from his thesis. Completed work and work in progress are described in the two following sections.

#### **Work completed**

##### **Core-Mantle Boundary Topography**

The broadest, most interdisciplinary research project carried out under this grant (and its predecessors) involves constraining the amplitude of the dynamically maintained topography at the core-mantle boundary (CMB). This is done by calculating the decade-

scale variation in length of day caused by coupling between CMB topography and the (variable) flow field in the core. This core flow is inferred from the secular variation of the magnetic field, while changes in length of day are determined using space geodesy. When the theory was developed a few years ago, we were limited by the availability of suitable models of core flow.

In the past year, we have used geostrophic core flow models provided by Jeremy Bloxham to compute core-mantle coupling for a variety of CMB topography models. An example is shown in Figure 1. The top frame is a CMB topography model estimated using seismic tomography by Gudmundsson (1989). The middle panel is a geostrophic core flow model from Bloxham (1989). The lower panel shows the contributions to the axial torque leading to changes in length of day. The total torque is a small number determined by the global integration of these nearly cancelling contributions.

We do not trust the models of either CMB topography or core flow sufficiently to have confidence in the details of this model. However, this type of model can still apply useful constraints to geophysical models of the CMB. Unless effective CMB topography is less than about 1/2 km in amplitude, the changes in length of day on decade timescales are larger than those observed. Such subdued CMB topography could result from a low-viscosity D" layer (Hager and Richards, 1989). These results are now written up (Hide et al, 1990) and will be submitted this month, as soon as drafting of figures is completed.

#### **Absolute value for mantle viscosity**

The dynamic geoid anomaly for a given internal density field depends on the relative variation of viscosity with depth, but does not depend on the absolute value of the viscosity, so long as the viscosity is high enough that inertial effects are negligible. The flow velocities, on the other hand, vary inversely as the absolute viscosity (Hager and Clayton, 1989). Our previous estimate of the absolute value of mantle viscosity was based on estimating the global heat flux advected by the flow driven by seismologically inferred mantle heterogeneities and assuming that this advected flux be a fraction of the observed surface flux.

Hager was fortunate to be invited, based on his geoid work, to give the Birch Lecture in Tectonophysics at the Fall, 1990 AGU (Hager, 1989). This led him to try to combine his early work on plate driving forces (Hager and O'Connell, 1981) with his later work on geoid anomalies in one internally consistent model.

The investigation of plate driving mechanisms involves using 3-D spherical flow models to calculate the driving tractions caused by relatively well constrained density contrasts in the mantle. These density contrasts originally included those due to the thermal structure of plates and subducted slabs, but now include those large-scale heterogeneities imaged using seismic tomography as well. Resisting tractions are due to viscous drag at the base of the plates (calculated in the flow models) and resistance at plate boundary faults. The extent to which the torques due to these tractions balance determines the success of the models. The drag at the base of the plates is directly proportional to mantle viscosity, providing a calibration not available from the geoid studies alone.

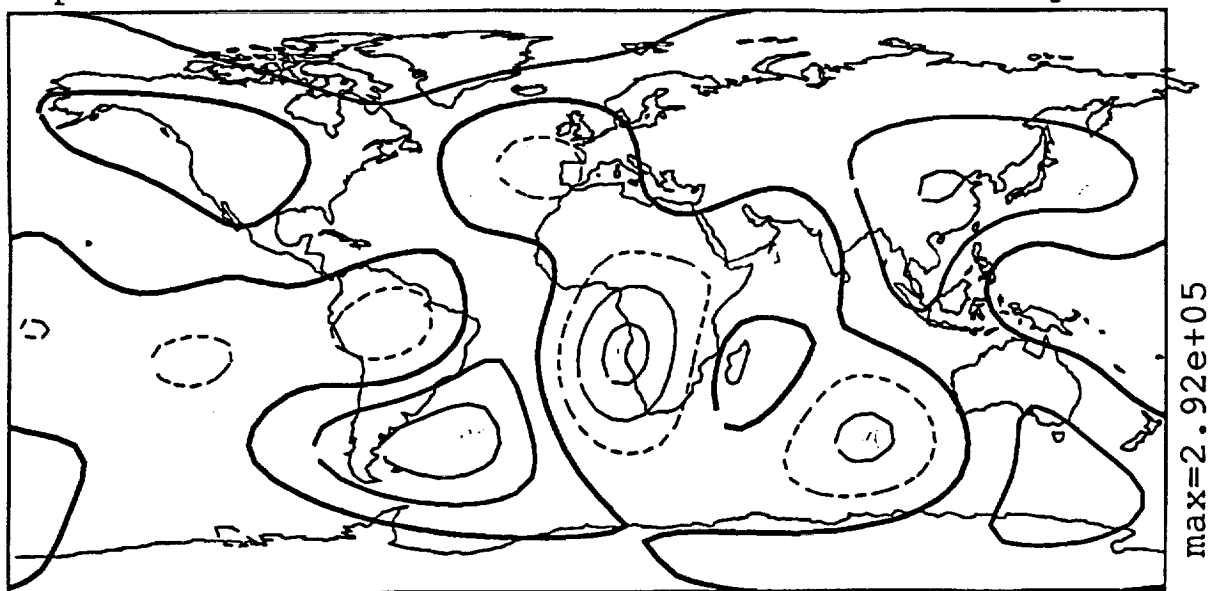
It is very encouraging to find that the new subducted slab model and the relative viscosity variation, based on geoid studies, provide an improved global torque balance. With a lower mantle viscosity of  $3 \times 10^{22}$  p, the driving and resisting torques match at the 80% variance reduction level. Figure 2 shows this match in a graphical way.

Figure 1: Top: CMB topography from Gudmundsson (1989). Depressed regions (e.g., Argentina) are shown in solid contours, uplifted regions (e.g., Africa) with an initial dashed contour. The zero contour is thick. The maximum uplift is 2.9 km.

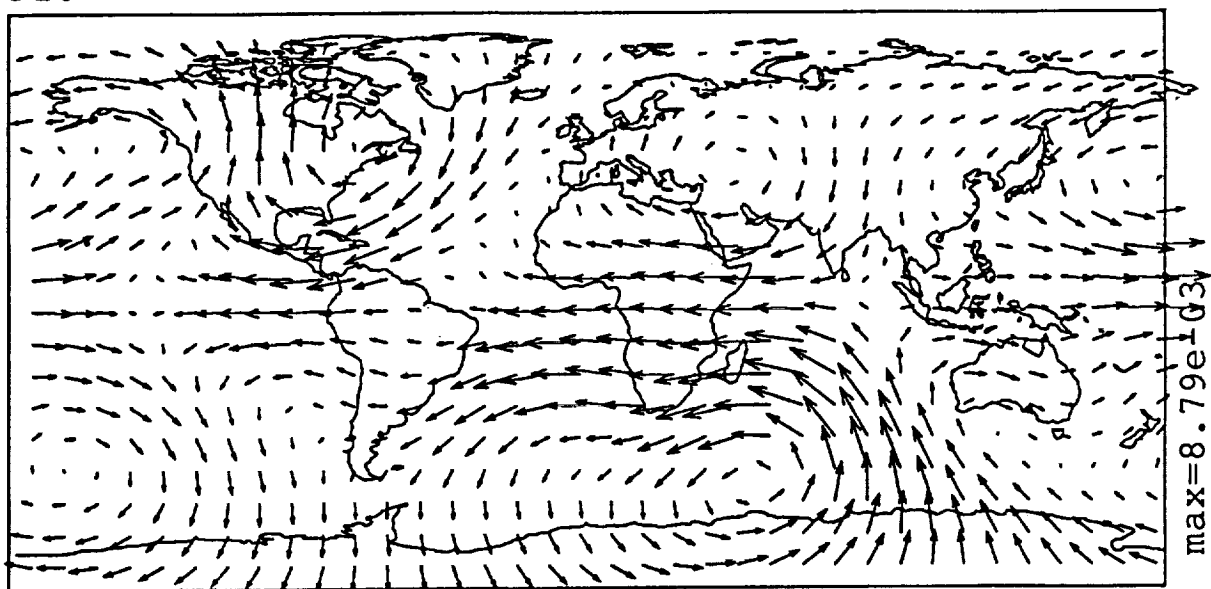
Center: A geostrophic model of flow at the top of the core (Bloxham, 1989). The maximum velocity is 0.09 mm/sec.

Bottom: Distribution of contributions to the axial torque. The heavy solid line is the zero contour. The many zero crossings show that the total torque is the result of the near cancellation of many counteracting contributions.

Topo



Flow



Torque Distribution



## BD + TF + CR + CC vs "Bndry Layer"

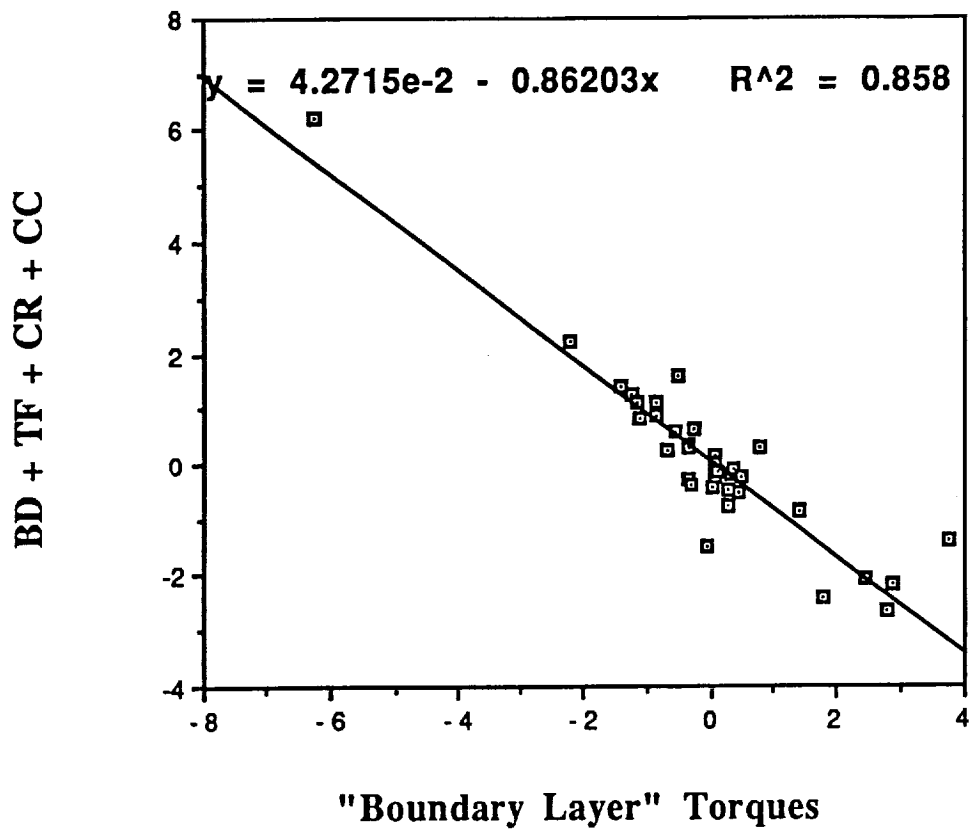


Figure 2: Torque balance for a global flow model. For each of the 12 major plates, the torques from Basal Drag (BD), Transform Fault resistance (TR), Collision Resistance at convergent margins (CR) and torques from the seismic heterogeneities inferred by Clayton and Comer (CC) are plotted vs the driving torques from density variations within the plates and subducted slabs ("Boundary Layer" Torques). The high correlation indicates that the torque balance is very good for this model.

Hager's model of mantle viscosity variation with depth is in accord with recent estimates from postglacial rebound studies (Nakada and Lambeck, 1989; Lambeck, personal communication). Hager has been invited to present his results at a conference in Erice, Italy this July devoted to the problem of determining mantle viscosity. Fortunately, his expenses are to be covered by the meeting organizers.

### **Code development**

Much of the work originally proposed addresses the effects of lateral variations in rheology, the effects of large amplitude deformation of a possible compositional boundary between the upper and lower mantles, and the effects of plate motions on the geoid. All this work requires as its foundation an efficient numerical code.

As part of his Ph. D. thesis research, graduate student Scott King has been developing a state-of-the-art finite element code for application to these and other problems in mantle convection (King and Hager, 1989; King et al, 1990). The code is highly vectorized for running on Cray class machines. It is now over three orders of magnitude faster than its parent code. It includes temperature and stress dependent rheology and the effects of compositional buoyancy. This code makes it possible to carry out much of the research proposed quite efficiently.

Another requirement for some of the research we plan is the ability to carry out numerical experiments in the spherical axisymmetric geometry appropriate for mantle plumes. During the past year, Caltech Postdoctoral Fellow Louise Kellogg, Scott King, and Hager have modified the finite element code to include spherical axisymmetry (Kellogg et al, 1989; 1990). This code will be used in modeling the geoid anomalies associated with plumes.

### **Lateral heterogeneity of subduction zone rheology**

The association of long-wavelength geoid highs with subducted slabs has been crucial in constructing our models of the viscosity structure of the mantle (Hager, 1984; Hager and Clayton, 1989; Hager and Richards, 1989). The main weakness of the geoid models up to now has been that they are based on the assumption of no lateral variations in viscosity - clearly a suspect assumption at subduction zones. We have begun a systematic investigation into the geophysical consequences of "realistic" (i.e., stress and temperature dependent) rheologies at subduction zones. As a first step in addressing the effects of lateral heterogeneity in effective viscosity on geoid anomalies at subduction zones, we investigated the effects of the parameterization of this region on plate and slab velocities in simple models of subduction (King and Hager, submitted, 1990; attached as Appendix B).

Convection with a Newtonian temperature-dependent rheology leads to little or no surface velocity unless zones of weakness are introduced. "Plate-like" features are observed in calculations both with Newtonian rheology, employing imposed weak zones, and with power-law (non-Newtonian) rheology, where high stresses at the trench reduce the effective viscosity. Since deformation at subduction zones involves faulting, both of these parameterizations should be treated with some skepticism. It is important to understand how the parameterizations affect the model results. We study the relationship between trench viscosity and plate velocity using a Newtonian rheology by varying the viscosity at the trench. The plate velocity is a function of the trench viscosity for fixed Rayleigh number and plate/slab viscosity. Slab velocities for non-Newtonian rheology calculations are significantly different from slab velocities from Newtonian rheology calculations at the same effective Rayleigh number. Both models give reasonable strain-rates for the slab when compared with estimates of seismic strain-rate. Non-Newtonian rheology eliminates

the need for imposed weak zones and provides a self-consistent fluid dynamical mechanism for subduction in numerical convection models.

### **Coolfont**

While not primarily research in itself, the planning for NASA's future embodied in the Coolfont meeting in July, 1990, is certainly important for carrying out the research program of the next decade. Hager participated vigorously in the "Coolfont Process," serving as a member of the Program Panel. In addition to the Coolfont meeting itself, this Panel held meetings both before and after Coolfont.

### **Work in progress**

As mentioned, Scott King is completing his Ph. D. Thesis this summer. The geophysical models in his thesis will concentrate on modeling the geoid anomalies associated with subducted slabs for a variety of structural models of the mantle. The effects addressed include those due to finite amplitude deformation of the 670 km discontinuity and lateral variations in viscosity.

### **Geoid anomalies for a chemically stratified mantle**

In our previous analytic models of the geoid, it is assumed that the dynamic topography associated with chemical discontinuities is small compared to the depth of the convecting layer. For models including a realistic chemical density contrast across the 670 km seismic discontinuity, this assumption will be violated (e.g., Hager, 1984; Hager and Clayton, 1989). King has developed a finite element code to address the importance of this effect in a series of numerical experiments.

Figure 3 shows the temperature and chemistry fields for one simple experiment formulated to address the consequences of deformation of the 670 km discontinuity on the geoid. In this uniform-viscosity model, flow is driven both by an imposed plate velocity on the "oceanic plate" and by thermal and compositional buoyancy. The density variations associated with composition are comparable to those due to temperature changes. Compositional differences are tracked using a marker chain approach (one of three options in the code).

The "670" is depressed substantially beneath the subducting slab and elevated substantially beneath the "ridge." There is little perturbation in its depth beneath the "continent" on the right hand side of the box.

The contributions to the geoid from different sources are shown in figure 4. The contribution from the temperature field is positive from the "subducted slab" and negative beneath the "ridge." That from the compositional buoyancy is just the opposite, positive where "lower mantle" comes closer to the surface beneath the spreading center and negative where it is depressed beneath the slab. The contribution from topography is somewhat complicated. There is a very long wavelength increase in elevation (and the corresponding geoid contribution) due to the pressure gradient associated with the kinematically driven flow. The sharp depression at the ridge is also a result of the kinematic boundary condition. The depression at the trench is from the dynamic topography due to the suction associated with the cold downwelling flow.

The total geoid is the sum of all these competing effects. As expected for uniform viscosity, there is a negative geoid anomaly associated with the cold downwelling. The "support" of the cold downwelling from the chemical barrier to flow is not sufficient to

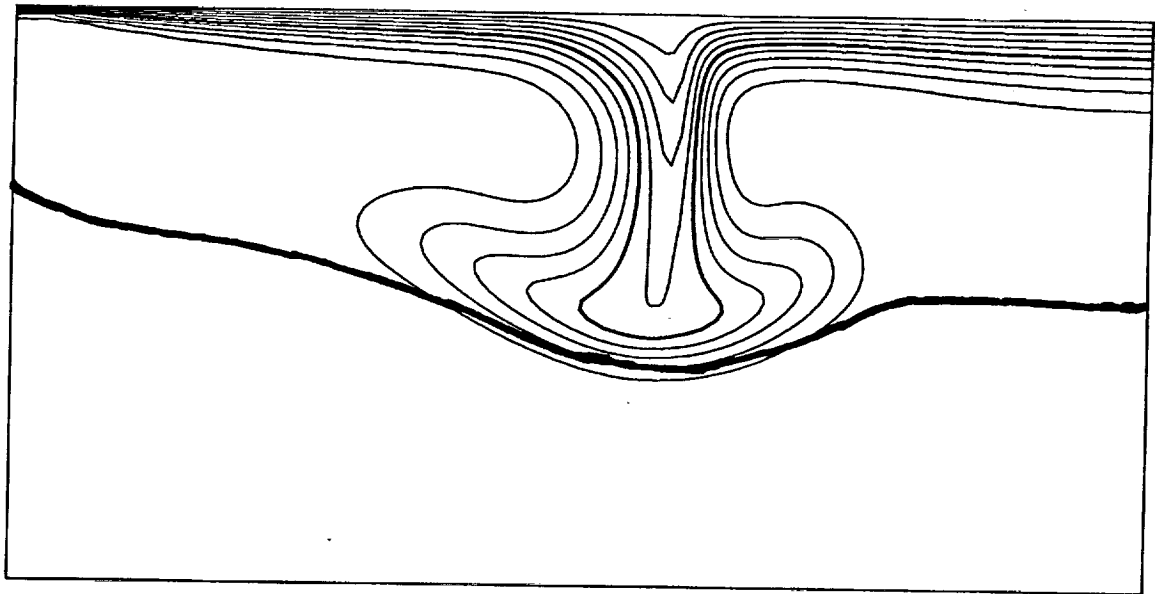


Figure 3: Isotherms and deflection of the "670 km discontinuity" (heavy line) for a model incorporating the dynamic effects of a difference in intrinsic density due to chemistry. The "oceanic plate" on the left half of the upper boundary of the box is driven to the right by an imposed boundary condition. The "continental plate" on the right side of the upper boundary is held fixed. The viscosity of the box is uniform.



### Geoid for a Stratified Mantle

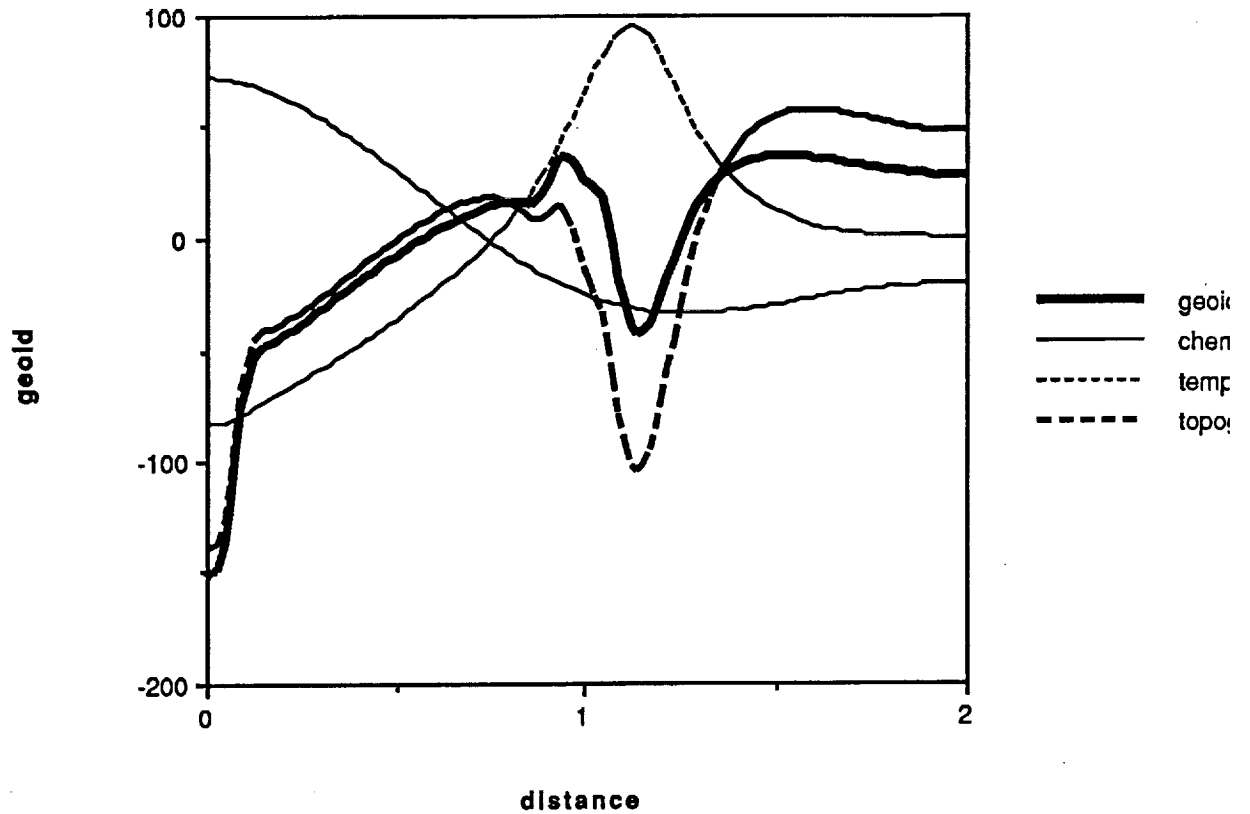


Figure 4: Individual contributions to the geoid calculated for the model shown in Figure 3. The total geoid is the sum of the contributions from deformation of the "670" (chemical), temperature variations, and dynamic topography.

reverse the sign of the geoid -- an increase in viscosity with depth is required. There are also profound effects due to the kinematic boundary conditions applied that need to be addressed, as discussed in the original proposal.

### **Geoid anomalies with lateral variations in viscosity**

We use the same algorithm used above to calculate geoid anomalies for finite element models incorporating realistic stress and temperature dependent rheologies. While the codes are debugged and running reliably, we do not expect to have geophysically interesting results until the end of the summer.

### **Publications**

While the research reported in them was carried out previous to this award period, three papers relevant to this grant were published during the past year. These are:

Constraints on the structure of mantle convection using seismic observations, flow models and the geoid, Bradford H. Hager and Robert W. Clayton, in W. R. Peltier, ed., *Mantle Convection*, Gordon and Breach, 657-763, 1989.

Long-wavelength variations in Earth's geoid: Physical models and dynamical implications, Bradford H. Hager and Mark A. Richards, *Phil. Trans. Roy. Soc. Lond. A.*, 328, 309-327, 1989.

Effects of long-wavelength lateral viscosity variations on the geoid, Mark A. Richards and Bradford H. Hager, *J. Geophys. Res.*, 94, 10,299-10,313, 1989.

In addition, two papers supported by this grant were submitted for publication:

Topographic core-mantle coupling and fluctuations in the Earth's rotation, R. Hide, R. W. Clayton, B. H. Hager, M. A. Spieth, and C.V. Voorhies, *Nature*, submitted, 1990.

The relationship between plate velocity and trench viscosity in Newtonian and power-law subduction calculations, Scott D. King and Bradford H. Hager, *Geophys. Res. Lett.*, submitted, 1990.

Abstracts supported by this grant are:

Dynamics and constitution of the Earth's interior, Bradford H. Hager, *EOS, Trans. Amer. Geophys. Union*, 70, 1334, 1989.

Slab rheology and the deflection of the 670 km discontinuity, Scott D. King and Bradford H. Hager, *EOS, Trans. Amer. Geophys. Union*, 70, 1314, 1989.

A finite element model of entrainment by axisymmetric plumes in a spherical shell, L. H. Kellogg, S. D. King, W. S. Kiefer, and B. H. Hager, *EOS, Trans. Amer. Geophys. Union*, 70, 1333, 1989.

Capacity of mantle plumes to carry material across compositional discontinuities, L. H. Kellogg, S.D. King, and B.H. Hager, *EOS, Trans. Amer. Geophys. Union*, 71, 527, 1990.